Developing the tools for "boosted frame" calculations.

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in collaboration with

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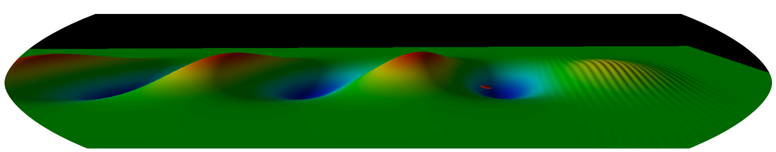
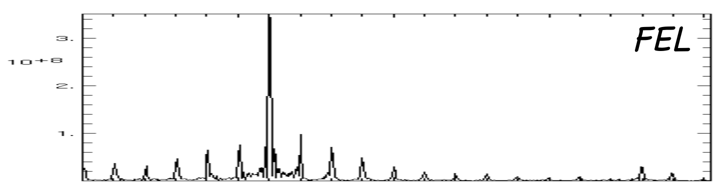
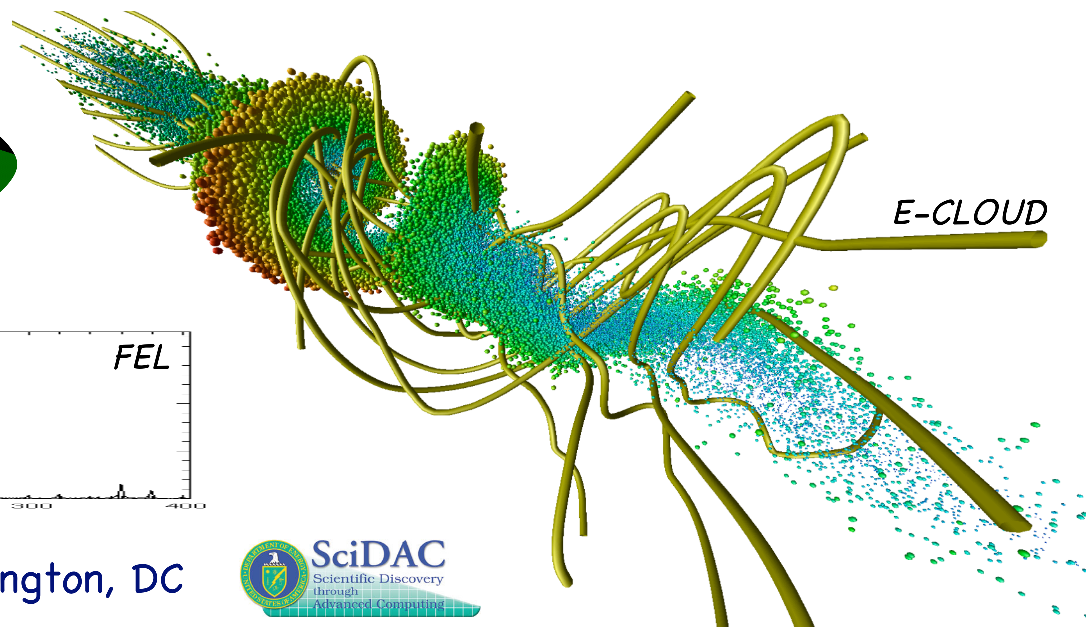


LWFA





E-CLOUD

SCIDAC Review, April 21-22, 2009, Washington, DC

Concept

- # of computational steps grows with the full range of space and time scales involved

- key observation
  - ***range*** of space and time scales is ***not*** a Lorentz invariant*

    scales as γ^2 in x and t

  - the optimum frame to minimize the range is not *necessarily* the lab frame

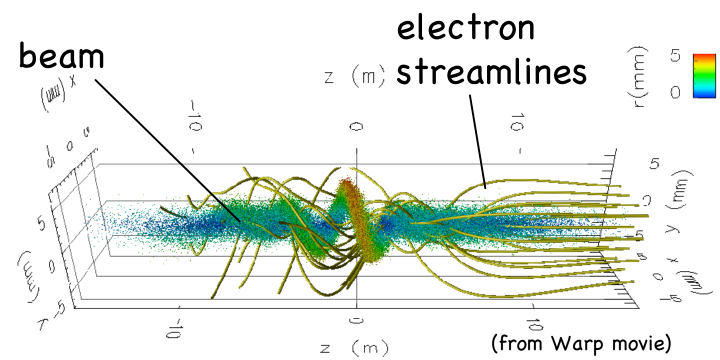
Choosing optimum frame of reference to minimize range can lead to dramatic speed-up for relativistic matter-matter or light-matter interactions.

*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)

Calculation of e-cloud induced TMC instability of a proton bunch

- Proton energy: γ=500 in Lab
- L= 5 km, continuous focusing

Code: Warp (Particle-In-Cell)



CPU time (2 quad-core procs):
- **lab frame: >2 weeks**
- **frame with γ^2=512: <30 min**

Speedup x1000

Seems simple but ⚠. Algorithms which work in one frame may break in another. Example: the Boris particle pusher.

- Boris pusher ubiquitous
  - In first attempt of e-cloud calculation using the Boris pusher, the beam was lost in a few betatron periods!
  - Position push: $\mathbf{X}^{n+1/2} = \mathbf{X}^{n-1/2} + \mathbf{V}^{n}\,\Delta t$ -- no issue
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \dfrac{q\,\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \dfrac{\gamma^{n+1}\mathbf{V}^{n+1} + \gamma^{n}\mathbf{V}^{n}}{2\,\gamma^{n+1/2}} \times \mathbf{B}^{n+1/2}\right)$

    issue: $\mathbf{E}+\mathrm{v}\times\mathbf{B}=0$ implies $\mathbf{E}=\mathbf{B}=0$ => large errors when $\mathbf{E}+\mathrm{v}\times\mathbf{B}\approx 0$ (e.g. relativistic beams).

- Solution
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^{n}\mathbf{V}^{n} + \dfrac{q\,\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \dfrac{\mathbf{V}^{n+1} + \mathbf{V}^{n}}{2} \times \mathbf{B}^{n+1/2}\right)$

- Not used before because of implicitness. We solved it analytically*

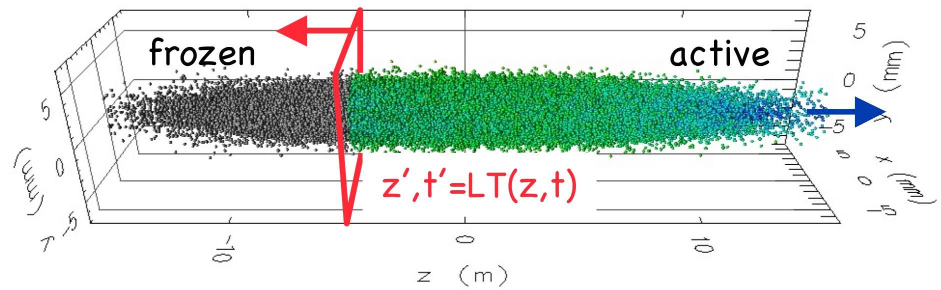
$$\begin{cases} \gamma^{i+1} = \sqrt{\dfrac{\sigma + \sqrt{\sigma^2 + 4(\tau^2 + u^{*2})}}{2}} \\ \mathbf{u}^{i+1} = [\mathbf{u}' + (\mathbf{u}' \cdot \mathbf{t})\mathbf{t} + \mathbf{u}' \times \mathbf{t}]/(1+t^2) \end{cases}$$

(with $\mathbf{u} = \gamma\mathbf{v}$, $\mathbf{u}' = \mathbf{u}^{i} + \dfrac{q\Delta t}{m}\left(\mathbf{E}^{i+1/2} + \dfrac{\mathbf{v}^{i}}{2} \times \mathbf{B}^{i+1/2}\right)$, $\boldsymbol{\tau} = (q\Delta t/2m)\mathbf{B}^{i+1/2}$, $u^{*} = \mathbf{u}' \cdot \boldsymbol{\tau}/c$, $\sigma = \gamma'^2 - \tau^2$, $\gamma' = \sqrt{1 + u'^2/c^2}$, $\mathbf{t} = \boldsymbol{\tau}/\gamma^{i+1}$).

*J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)

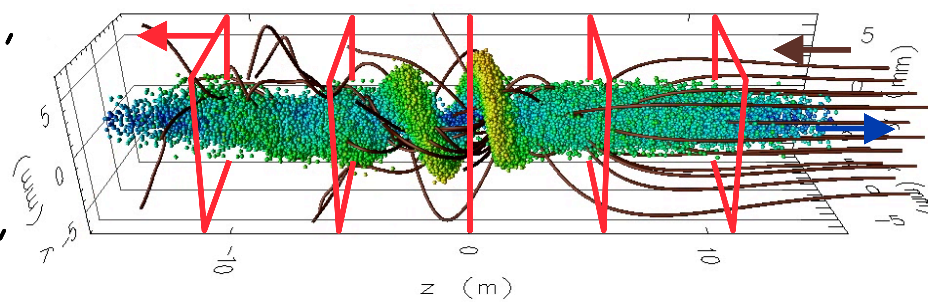
Other complication: input/output

- Often, initial conditions known and output desired in laboratory frame
  - relativity of simultaneity => inject/collect at plane(s) ⊥ to direction of boost.
- Injection through a moving plane in boosted frame (fix in lab frame)
  - fields include frozen particles,
  - same for laser in EM calculations.



- Diagnostics: collect data at a collection of planes
  - fixed in lab fr., moving in boosted fr.,
  - interpolation in space and/or time,
  - already done routinely with Warp

  for comparison with experimental data, often known at given stations in lab.

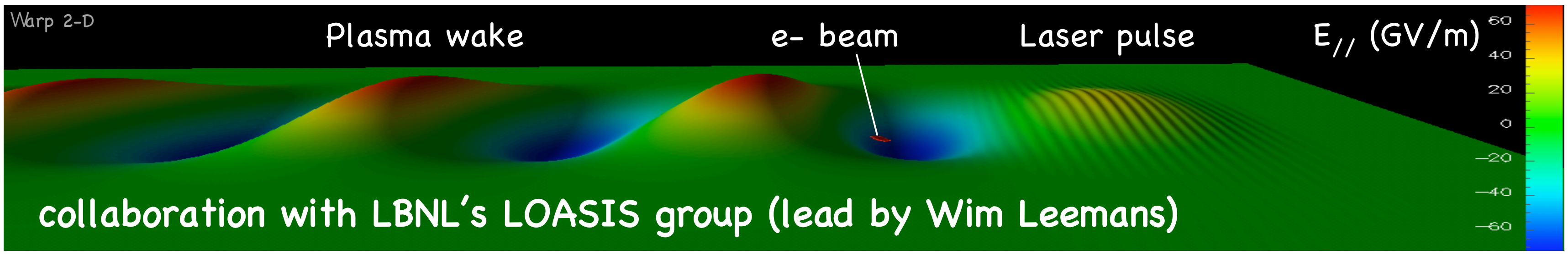


Application to Laser-plasma wakefield accelerators

- New electromagnetic solver implemented in Warp (SBIR funding)
  - scaling test (3-D decomp)

# procs	256 (8x8x4)	512 (8x8x8)	1024 (8x8x16)
# cell, particles	$1{,}024^2$x512, 100M	$1{,}024^3$, 200M	$1{,}024^2$x2,048, 400M
Time ratio	1.	1.04	1.12

- Applied to modeling of one stage of LWFA (2-D for now, 3-D to follow)



Average beam energy and CPU time vs position in lab frame